Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Paper reference **8FM0/21**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Q:1/1/1/

P 6 8 6 1 7 A 0 1 1 6

Pearson

1. Use algebra to find the set of values of x for which

$$x \geqslant \frac{2x + 15}{2x + 3}$$

(6)

$x(2x+3)^2 \geqslant (2x+15)(2x+3) \leftarrow$ multiply by $(2x+3)^2$ to ensure that it's positive, because a negative would flip the inequality sign

$x(4x^2+12x+9) \geqslant 4x^2+36x+45$ (1)

$4x^3+12x^2+9x \geqslant 4x^2+36x+45$

$4x^3+8x^2-27x-45 \geqslant 0$

$(2x+3)(2x^2+3x-2x-15) \geqslant 0$

$(2x+3)(x+3)(2x-5) \geqslant 0$ (1)

$\therefore$ critical values are $x=-3$, $x=\frac{5}{2}$, $x=-\frac{3}{2}$ (1)

When $x < -3$: $-4 \geqslant \frac{2(-4)+15}{2(-4)+3} \Rightarrow -4 \geqslant -14$ ✗

When $-3 < x < -\frac{3}{2}$: $-2 \geqslant \frac{2(-2)+15}{2(-2)+3} \Rightarrow -2 \geqslant -11$ ✓

When $-\frac{3}{2} < x < \frac{5}{2}$: $0 \geqslant \frac{2(0)+15}{2(0)+3} \Rightarrow 0 \geqslant 5$ ✗

When $x > \frac{5}{2}$: $3 \geqslant \frac{2(3)+15}{2(3)+3} \Rightarrow 3 \geqslant \frac{7}{3}$ ✓

$\therefore \{x \in \mathbb{R} : -3 \leq x < -\frac{3}{2},\ x \geqslant \frac{5}{2}\}$ (1) (1) (1)

$2x+3 \neq 0$ because this would invalidate original inequality.

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Question 1 continued

(Total for Question 1 is 6 marks)

PMT

2. A population of deer was introduced onto an island.

The number of deer, P, on the island at time t years following their introduction is modelled by the differential equation

$$\frac{dP}{dt} = \frac{P}{5000}\left(1000 - \frac{P(t+1)}{6t+5}\right) \qquad t > 0$$

It was estimated that there were 540 deer on the island six months after they were introduced.

Use **two** applications of the approximation formula $\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_n}{h}$ to estimate the number of deer on the island 10 months after they were introduced.

(7)

$t_0 = \frac{6}{12}$ ← t is in years and the first step is 6 months ($\frac{1}{2}$ a year).

then $t_1 = \frac{8}{12}$, $t_2 = \frac{10}{12}$

$h = \frac{10-6}{12} \div 2 = \frac{2}{12}$ 1 ← '$\div 2$' for no. of steps

First application:

$$\left(\frac{dP}{dt}\right)_0 = \frac{540}{5000}\left[1000 - \frac{540 \times (\frac{6}{12}+1)}{6 \times \frac{6}{12} + 5}\right] = 97.605 \quad 1$$

$$97.605 \approx \frac{P_1 - 540}{1/6} \quad 1 \quad \Rightarrow \quad P_1 \approx 556.2675 \quad 1$$

Second application.

$$\left(\frac{dP}{dt}\right)_1 = \frac{556.2}{5000}\left[1000 - \frac{556.2 \times (\frac{8}{12}+1)}{6 \times \frac{8}{12} + 5}\right] = 99.793 \quad 1$$

$$99.793 \approx \frac{P_2 - 556.2}{1/6} \quad \Rightarrow \quad P_2 \approx 572.8.. \quad 1$$

∴ There are approximately 573 deer after 10 months 1

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Question 2 continued

(Total for Question 2 is 7 marks)

3. (a) Use $t = \tan\frac{\theta}{2}$ to show that, where both sides are defined

$$\frac{29 - 21\sec\theta}{20 - 21\tan\theta} \equiv \frac{5t + 2}{2t + 5}$$

(4)

(b) Hence, again using $t = \tan\frac{\theta}{2}$, prove that, where both sides are defined

$$\frac{20 + 21\tan\theta}{29 + 21\sec\theta} \equiv \frac{29 - 21\sec\theta}{20 - 21\tan\theta}$$

(3)

(a) Using t-formulae : $\cos\theta = \frac{1-t^2}{1+t^2}$

$\tan\theta = \frac{2t}{1-t^2}$ $\quad \therefore \sec\theta = \frac{1+t^2}{1-t^2}$ 1 $\quad \sec = \frac{1}{\cos}$

$$\frac{29-21\sec\theta}{20-21\tan\theta} = \frac{29-21\left(\frac{1+t^2}{1-t^2}\right)}{20-21\left(\frac{2t}{1-t^2}\right)} \quad 1$$

$\times(1-t^2)$ to all terms

$$= \frac{29(1-t^2)-21(1+t^2)}{20(1-t^2)-21(2t)}$$

$$= \frac{8-50t^2}{-20t^2-42t+20} \quad 1$$

$$= \frac{-\cancel{2(5t-2)}(5t+2)}{-\cancel{2(5t-2)}(2t+5)}$$

$$= \frac{5t+2}{2t+5} \quad 1$$

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Question 3 continued

(b) $$\frac{20+21\tan\theta}{29+21\sec\theta} = \frac{20+21\left(\frac{2t}{1-t^2}\right)}{29+21\left(\frac{1+t^2}{1-t^2}\right)}$$

$$= \frac{20(1-t^2)+21(2t)}{29(1-t^2)+21(1+t^2)} \quad 1$$

$$= \frac{20+42t-20t^2}{50-8t^2}$$

$$= \frac{\cancel{-2}(5t+2)\cancel{(2t-5)}}{\cancel{-2}(2t+5)\cancel{(2t-5)}} \quad 1$$

$$= \frac{5t+2}{2t+5}$$

$\therefore$ using the solution from part (a), $\dfrac{20+21\tan\theta}{29+21\sec\theta} \equiv \dfrac{29-21\sec\theta}{20-21\tan\theta}$ 1

(Total for Question 3 is 7 marks)

4. The parabola C has equation $y^2 = 10x$

The point F is the focus of C.

(a) Write down the coordinates of F.

(1)

The point P on C has y coordinate q, where $q > 0$

(b) Show that an equation for the tangent to C at P is given by

$$10x - 2qy + q^2 = 0$$

(3)

The tangent to C at P intersects the directrix of C at the point A.

The point B lies on the directrix such that PB is parallel to the x-axis.

(c) Show that the point of intersection of the diagonals of quadrilateral $PBAF$ always lies on the y-axis.

(5)

(a) $\left(\frac{5}{2}, 0\right)$ (1) $\leftarrow$ put into form $y^2 = 4ax$: $y^2 = 4\left(\frac{10}{4}\right)x$
then focus $= (a, 0)$ $\therefore \left(\frac{5}{2}, 0\right)$

(b) $\frac{dy}{dx} = \frac{2 \times \frac{5}{2}}{q} = \frac{5}{q}$ (1) $\leftarrow$ $\frac{d}{dx}(y^2 = 4ax) = \frac{2a}{y}$

At P, $x = \frac{q^2}{10}$. $\leftarrow$ $y^2 = 10x \rightarrow x = \frac{y^2}{10}$ | $y = q$

$y - q = \frac{5}{q}\left(x - \frac{q^2}{10}\right)$ (1) $\leftarrow$ $y - y_1 = m(x - x_1)$

$y - q = \frac{5x}{q} - \frac{5q}{10}$ $\times q$

$qy - q^2 = 5x - \frac{q^2}{2}$ $\times 2$

$2qy - 2q^2 = 10x - q^2$

$10x - 2qy + q^2 = 0$ (1)

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Question 4 continued

(c) $B\left(-\frac{5}{2}, q\right)$ 1 ← directrix has equation $x + \frac{5}{2} = 0$

BF: $\frac{y-0}{q-0} = \frac{x - \frac{5}{2}}{-\frac{5}{2} - \frac{5}{2}}$ ← or use $y - y_1 = m(x - x_1)$

$$y = q\left(\frac{x - \frac{5}{2}}{-5}\right)$$

$$y = -\frac{q}{5}\left(x - \frac{5}{2}\right)$$ 1 ← rearrange to put in terms of y..

AP is a tangent so diagonals meet when

$$10x - 2q\left[-\frac{q}{5}\left(x - \frac{5}{2}\right)\right] + q^2 = 0$$ 1 ← then substitute into equation from (b)

$$10x + \frac{2q^2}{5}\left(x - \frac{5}{2}\right) + q^2 = 0$$

$$10x + \frac{2q^2x}{5} - \frac{10q^2}{10} + q^2 = 0$$

$$x\left(10 + \frac{2q^2}{5}\right) - q^2 + q^2 = 0$$

$$x\left(10 + \frac{2q^2}{5}\right) = 0$$ 1 ← if $q < 0$, $q^2 > 0$ and if $q \geq 0$, $q^2 \geq 0$ $\therefore 10 + \frac{2q^2}{5} > 0$

$\therefore 10 + \frac{2q^2}{5}$ will be greater than 0 for all $q \in \mathbb{R}$, therefore $x = 0$ 1

and the intersection lies on the y-axis

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5.

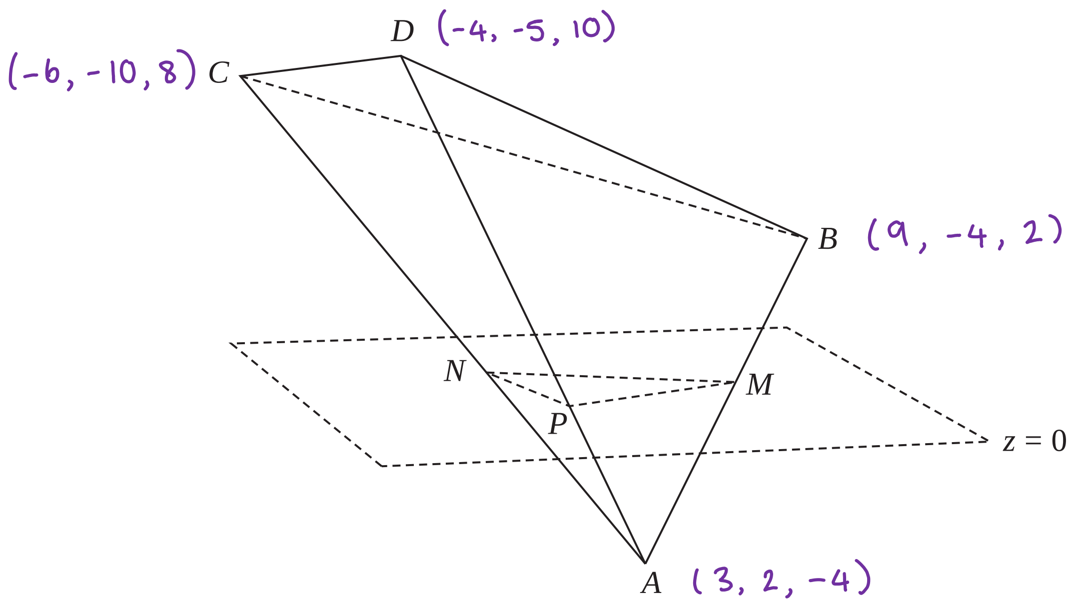


Figure 1

The points $A(3, 2, -4)$, $B(9, -4, 2)$, $C(-6, -10, 8)$ and $D(-4, -5, 10)$ are the vertices of a tetrahedron.

The plane with equation $z = 0$ cuts the tetrahedron into two pieces, one on each side of the plane.

The edges AB, AC and AD of the tetrahedron intersect the plane at the points M, N and P respectively, as shown in Figure 1.

Determine

(a) the coordinates of the points M, N and P, **(3)**

(b) the area of triangle MNP, **(2)**

(c) the exact volume of the solid $BCDPNM$. **(6)**

(a) $\overrightarrow{AB} = B - A = \begin{pmatrix} 9 \\ -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix}$

$\overrightarrow{OM} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \frac{4}{6}\begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \\ 0 \end{pmatrix}$ ① $\therefore M = (7, -2, 0)$

$z = 0$ is $\frac{4}{6}$ of the way between

$A_z = -4$ and $B_z = 2$ in the direction $\overrightarrow{AB}$

Question 5 continued

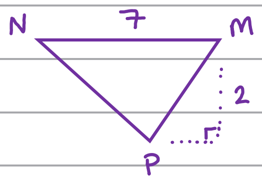
$$\overrightarrow{AC} = C - A = \begin{pmatrix} -6 \\ -10 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -9 \\ -12 \\ 12 \end{pmatrix}$$

$$\overrightarrow{ON} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \frac{4}{12}\begin{pmatrix} 9 \\ -12 \\ 12 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} \quad \therefore N = (0, -2, 0) \; 1$$

$$\overrightarrow{AD} = D - A = \begin{pmatrix} -4 \\ -5 \\ 10 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 14 \end{pmatrix}$$

$$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \frac{4}{14}\begin{pmatrix} -7 \\ -7 \\ 14 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \therefore P = (1, 0, 0) \; 1$$

(b) $\overrightarrow{MN} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \\ 0 \end{pmatrix}$ so $\overrightarrow{MN}$ is parallel to the x-axis with length 7.



P is on y-axis and M + N are at $y = -2$,
$\therefore$ height $= 2$ and they form a triangle
1 for recognising $b = 7$, $h = 2$

$$\text{Area} = \frac{1}{2} \times b \times h = \frac{1}{2} \times 7 \times 2 = 7 \; 1$$

(c) $\text{Vol}(NPMA) = \frac{1}{3} \times \text{area} \times h$ 1

$h = 0 - (A_z) = 0 - (-4) = 4$ ← MNP is on $z = 0$

$$\text{Vol}(NPMA) = \frac{1}{3} \times 7 \times 4 = \frac{28}{3} \; 1$$

Question 5 continued

$$\text{Vol (ABCD)} = \frac{1}{6}\left|\vec{AB}\cdot(\vec{AC}\times\vec{AD})\right|$$

$$= \frac{1}{6}\left|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\left(\begin{pmatrix}-3\\-4\\4\end{pmatrix}\times\begin{pmatrix}-2\\-2\\4\end{pmatrix}\right)\right| \quad 1$$

$$= \frac{1}{6}\left|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\begin{pmatrix}-84\\42\\-21\end{pmatrix}\right| \leftarrow 6\times-84+-6\times42+6\times-21$$

$$= \frac{1}{6}\left|-882\right|$$

$$= 147 \quad 1$$

$$\underset{\substack{\uparrow\\\text{whole shape}}}{\text{Vol (ABCD)}} - \underset{\substack{\uparrow\\\text{smaller shape}}}{\text{Vol (APNM)}} = 147 - \frac{28}{3} \; 1 \; = \frac{413}{3} \quad 1$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS